Paper Reference(s)

6678

Edexcel GCE

Mechanics M2

Advanced Level

Friday 28 January 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A cyclist starts from rest and moves along a straight horizontal road. The combined mass of the cyclist and his cycle is 120 kg. The resistance to motion is modelled as a constant force of magnitude 32 N. The rate at which the cyclist works is 384 W. The cyclist accelerates until he reaches a constant speed of v m s^{-1}.

Find

(*a*) the value of v,

(3)

(*b*) the acceleration of the cyclist at the instant when the speed is 9 m s^{-1}.

(3)

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2. A particle of mass 2 kg is moving with velocity $(5\mathbf{i} + \mathbf{j})$ m s^{-1} when it receives an impulse of $(-6\mathbf{i} + 8\mathbf{j})$ N s. Find the kinetic energy of the particle immediately after receiving the impulse.

(5)

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3. A particle moves along the x-axis. At time $t = 0$ the particle passes through the origin with speed 8 m s^{-1} in the positive x-direction. The acceleration of the particle at time t seconds, $t \geq 0$, is $(4t^3 - 12t)$ m s^{-2} in the positive x-direction.

Find

(*a*) the velocity of the particle at time t seconds,

(3)

(*b*) the displacement of the particle from the origin at time t seconds,

(2)

(*c*) the values of t at which the particle is instantaneously at rest.

(3)

4.

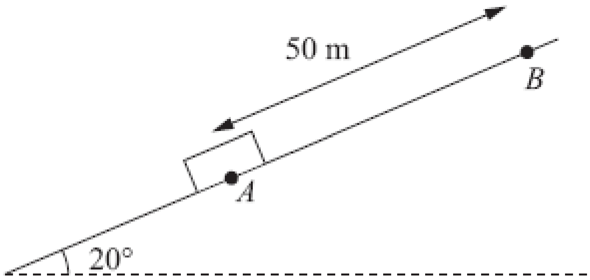

Figure 1

A box of mass 30 kg is held at rest at point *A* on a rough inclined plane. The plane is inclined at 20° to the horizontal. Point *B* is 50 m from *A* up a line of greatest slope of the plane, as shown in Figure 1. The box is dragged from *A* to *B* by a force acting parallel to *AB* and then held at rest at *B*. The coefficient of friction between the box and the plane is $\frac{1}{4}$. Friction is the only non-gravitational resistive force acting on the box. Modelling the box as a particle,

(*a*) find the work done in dragging the box from *A* to *B*.

(6)

The box is released from rest at the point *B* and slides down the slope. Using the work-energy principle, or otherwise,

(*b*) find the speed of the box as it reaches *A*.

(5)

5.

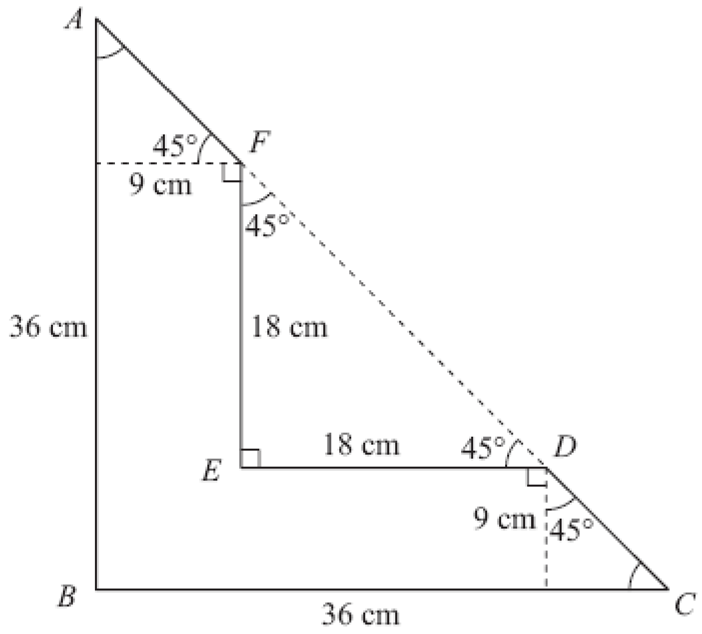

Figure 2

The uniform L-shaped lamina *ABCDEF*, shown in Figure 2, has sides *AB* and *FE* parallel, and sides *BC* and *ED* parallel. The pairs of parallel sides are 9 cm apart. The points *A*, *F*, *D* and *C* lie on a straight line.

$AB = BC = 36$ cm, $FE = ED = 18$ cm.

$\angle ABC = \angle FED = 90°$, and $\angle BCD = \angle EDF = \angle EFD = \angle BAC = 45°$.

(*a*) Find the distance of the centre of mass of the lamina from

(i) side *AB*,

(ii) side *BC*.

(7)

The lamina is freely suspended from *A* and hangs in equilibrium.

(*b*) Find, to the nearest degree, the size of the angle between *AB* and the vertical.

(3)

6. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

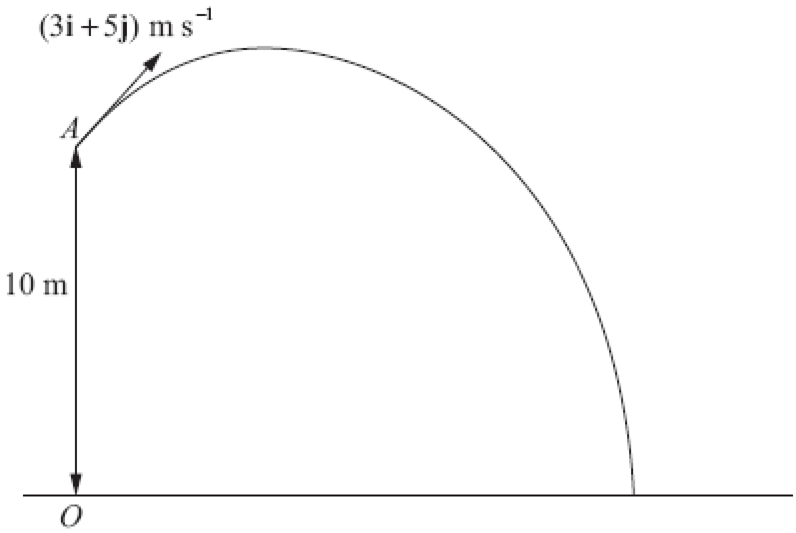

Figure 3

At time $t = 0$, a particle P is projected from the point A which has position vector $10\mathbf{j}$ metres with respect to a fixed origin O at ground level. The ground is horizontal. The velocity of projection of P is $(3\mathbf{i} + 5\mathbf{j})$ m s^{-1}, as shown in Figure 3. The particle moves freely under gravity and reaches the ground after T seconds.

(*a*) For $0 \leq t \leq T$, show that, with respect to O, the position vector, $\mathbf{r}$ metres, of P at time t seconds is given by

$$\mathbf{r} = 3t\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$$

(3)

(*b*) Find the value of T.

(3)

(*c*) Find the velocity of P at time t seconds $(0 \leq t \leq T)$.

(2)

When P is at the point B, the direction of motion of P is 45° below the horizontal.

(*d*) Find the time taken for P to move from A to B.

(2)

(*e*) Find the speed of P as it passes through B.

(2)

7.

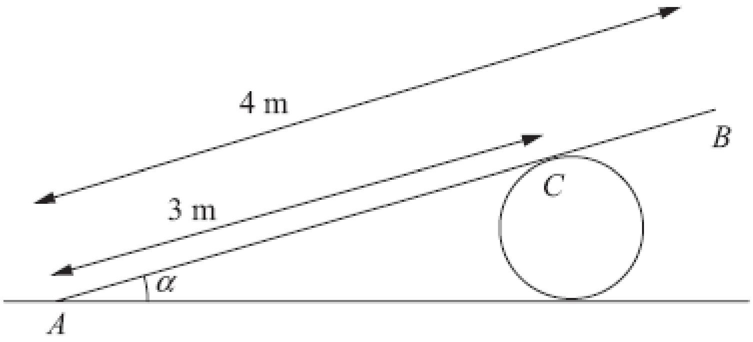

Figure 4

A uniform plank *AB*, of weight 100 N and length 4 m, rests in equilibrium with the end *A* on rough horizontal ground. The plank rests on a smooth cylindrical drum. The drum is fixed to the ground and cannot move. The point of contact between the plank and the drum is *C*, where $AC = 3$ m, as shown in Figure 4. The plank is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle α to the horizontal, where $\sin\alpha = \frac{1}{3}$. The coefficient of friction between the plank and the ground is μ.

Modelling the plank as a rod, find the least possible value of μ.

(10)

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8. A particle *P* of mass *m* kg is moving with speed 6 m s^{-1} in a straight line on a smooth horizontal floor. The particle strikes a fixed smooth vertical wall at right angles and rebounds. The kinetic energy lost in the impact is 64 J. The coefficient of restitution between *P* and the wall is $\frac{1}{3}$.

(*a*) Show that $m = 4$.

(6)

After rebounding from the wall, *P* collides directly with a particle *Q* which is moving towards *P* with speed 3 m s^{-1}. The mass of *Q* is 2 kg and the coefficient of restitution between *P* and *Q* is $\frac{1}{3}$.

(*b*) Show that there will be a second collision between *P* and the wall.

(7)

TOTAL FOR PAPER: 75 MARKS

END

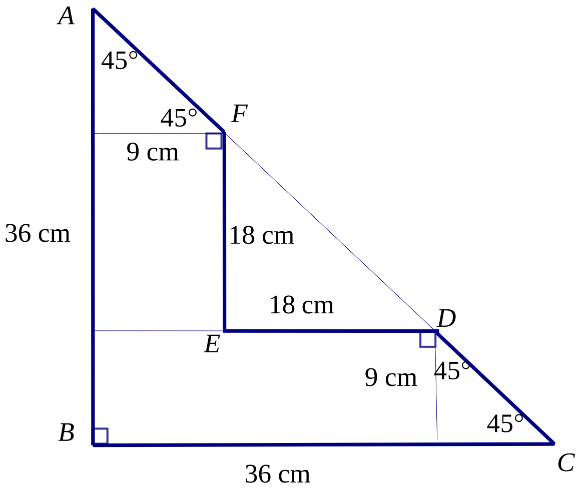
January 2011
Mechanics M2 6678
Mark Scheme

Question Number	Scheme	Marks
1. **(a)**	Constant speed $\Rightarrow$ Driving force = resistance , $F = 32$. $P = F \times v = 32v = 384$ $v = 12 \ (\text{ms}^{-1})$	B1 M1 A1 (3)
(b)	$P = F \times v \Rightarrow \ 384 = F \times 9, \ F = \frac{384}{9}$ Their $F - 32 = 120a$, $a = 0.089\left(\text{ms}^{-2}\right)$	M1 M1 A1 (3) **[6]**
2.	$\mathbf{I} = (\mathbf{-6i} + \mathbf{8j}) = \mathbf{2}(\mathbf{v} - (\mathbf{5i} + \mathbf{j}))$ $-3\mathbf{i} + 4\mathbf{j} = \mathbf{v} - 5\mathbf{i} - \mathbf{j}$ $\mathbf{v} = 2\mathbf{i} + 5\mathbf{j}$ $\text{KE} = \frac{1}{2} \times 2 \times \lvert v \rvert^2 = \left(\sqrt{2^2 + 5^2}\right)^2 = 29 \ (\text{J})$	M1A1 A1 M1 A1 **[5]**
3. **(a)**	$a = 4t^3 - 12t$ Convincing attempt to integrate $v = t^4 - 6t^2 \ (+c)$ Use initial condition to get $v = t^4 - 6t^2 + 8\left(\text{ms}^{-1}\right)$.	M1 A1 A1 (3)
(b)	Convincing attempt to integrate $s = \frac{t^5}{5} - 2t^3 + 8t \ (+0)$ Integral of their v	M1 A1ft (2)
(c)	Set their $v = 0$ Solve a quadratic in t^2 $(t^2 - 2)(t^2 - 4) = 0 \Rightarrow$ at rest when $t = \sqrt{2}, \ t = 2$	M1 DM1 A1 (3) **[8]**

Question Number	Scheme	Marks
4. **(a)**	Work done against friction $= 50\times\mu R$	M1
	$= 50\times ¼\times 30\cos 20°\times 9.8$	A1
	Gain in GPE $= 30\times 9.8\times 50\sin 20°$	M1 A1
	Total work done = WD against Friction + gain in GPE	DM1
	$= 8480(\text{J}),\ 8500(\text{J})$	A1
		(6)
(b)	Loss in GPE = WD against friction + gain in KE 3 terms	M1
	$30\times 9.8\times 50\sin 20° = 50\times ¼\times 30\times 9.8\times\cos 20° + ½\times 30\times v^2$ -1 ee	A2,1,0
	$½\ v^2 = 50\times 9.8\times(\sin 20° - ¼\cos 20°)$,	DM1
	$v = 10.2\ \text{m s}^{-1}$.	A1
		(5) **[11]**

5.

(a)



Divide the shape into usable areas, e.g.:

Shape	C of mass	Units of mass
Rectangle 27 x 9	(13.5,4.5)	243 (6)
Right hand triangle	(30,3)	40.5 (1)
Top triangle	(3,30)	40.5 (1)
Rectangle 9 x 18	(4.5,18)	162 (4)

Mass ratios — B1
Centres of mass — B1

Take moments about AB: — M1
$6\times 13.5+1\times 30+4\times 4.5+1\times 3=132=12\bar{x}$, — A(2,1,0)

$\bar{x}=11\,(\text{cm})$ solve for *x* (or *y*) co-ord — A1
$\bar{y}=11\,(\text{cm})$ using the symmetry — B1ft

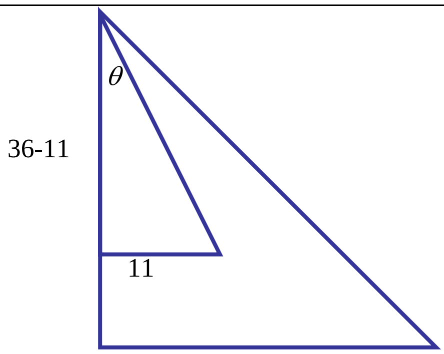
Alternative:

Shape	C of mass	Units of mass
Small triangle	(12,12)	.5 x 18x 18
Large triangle	(15,15)	.5 x 36 x 36

$$\frac{1}{2}\times 36\times 36\times 12-\frac{1}{2}\times 18\times 18\times 15=\frac{1}{2}(36\times 36-18\times 18)\bar{x}\text{ etc.}$$

(7)

(b)



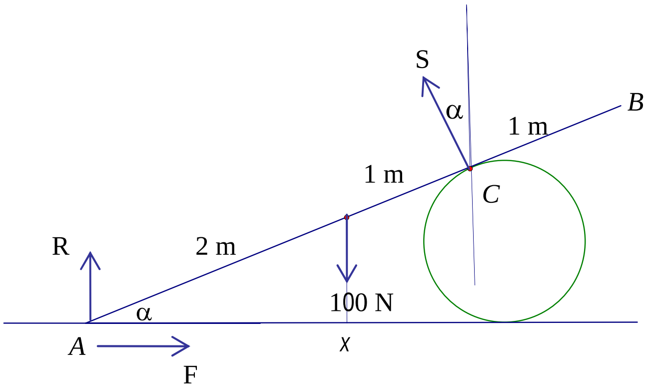
$\tan\theta=\dfrac{\bar{x}}{36-\bar{y}}$ — M1

$\tan\theta=\dfrac{11}{25}=0.44$ — A1ft

$\theta=24°$ — A1

(3)
[10]

6. **(a)**	Using $s = ut + \frac{1}{2}at^2$ Method must be clear $\mathbf{r} = (3t)\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$ Answer given	M1 A1 A1 (3)
(b)	$\mathbf{j}$ component $= 0$: $10 + 5t - 4.9t^2$ quadratic formula: $t = \frac{5 \pm \sqrt{25 + 196}}{9.8} = \frac{5 \pm \sqrt{221}}{9.8}$ T = 2.03(s), 2.0 (s) positive solution only.	M1 DM1 A1 (3)
(c)	Differentiating the position vector (or working from first principles) $\mathbf{v} = 3\mathbf{i} + (5 - 9.8t)\mathbf{j}$ (ms^{-1})	M1 A1 (2)
(d)	At B the **j** component of the velocity is the negative of the **i** component: $5 - 9.8t = -3$, $8 = 9.8t$, $t = 0.82$	M1 A1 (2)
(e)	$\mathbf{v} = 3\mathbf{i} - 3\mathbf{j}$, speed = $\sqrt{3^2 + 3^2} = \sqrt{18} = 4.24$ (m s^{-1})	M1A1 (2) **[12]**

Question Number	Scheme	Marks
7.		
	Taking moments about A: $3S = 100 \times 2 \times \cos\alpha$	M1 A1
	Resolving vertically: $R + S\cos\alpha = 100$	M1 A1
	Resolving horizontally: $S\sin\alpha = F$	M1 A1
	(Most alternative methods need 3 independent equations, each one worth M1A1. Can be done in 2 e.g. if they resolve horizontally and take moments about X then $R \times 2 \times \cos\alpha = S \times (3 - 2 \times \cos^2\alpha)$ scores M2A2)	
	Substitute trig values to obtain correct values for F and R (exact or decimal equivalent).	DM1
	$\left(S = \frac{200\sqrt{8}}{9}\right)$, $R = 100 - \frac{1600}{27} = \frac{1100}{27} \approx 40.74$, $F = \frac{200\sqrt{8}}{27} \approx 20.95...$	A1
	$F \le \mu R,\ 200\sqrt{8} \le \mu \times 1100,\ \ \mu \ge \frac{200\sqrt{8}}{1100} = \frac{2\sqrt{8}}{11}$.	M1
	Least possible μ is 0.514 (3sf), or exact.	A1
		[10]

Question Number	Scheme	Marks
8.		
(a)	KE lost : $\frac{1}{2}\times m\times 36-\frac{1}{2}\times m\times v^2=64$	M1A1
	Restitution: $v=$ 1/3 x 6 = 2	M1A1
	Substitute and solve for m: $\frac{1}{2}\times m\times 36-\frac{1}{2}\times m\times 4=64=16m$	DM1
	m = 4 answer given	A1
		(6)
(b)		
	Conservation of momentum: $6-8=4w-2v$ their "2"	M1A1ft
	Restitution: $v+w=⅓(2\ +\ 3)$ their "2"	M1A1ft
	$v=\frac{5}{3}-w$	
	Solve for w: $-2=4w-2(\frac{5}{3}-w)=6w-\frac{10}{3}$	DM1
	$\frac{4}{3}=6w$	A1
	$\left(w=4/18=2/9\,\text{m s}^{-1}\right)$	
	$w>\ 0\ \Rightarrow$ will collide with the wall again	A1
		(7)
		[13]